Name————————————, ID ——————

 

# EXAM II, MTH 512, Spring 2015

Ayman Badawi

**QUESTION 1.** (i) Let $<,>$ be an inner product on a vector space $V$. Prove that $||v + u|| \leq ||v|| + ||u||$ for every $u, v \in V$.

(ii) Let $A$ be an invertible $n \times n$ matrix. Prove that $deg(m_{A^{-1}}(x)) = deg(m_A(x))$ (i.e., show that the degree of the minimum polynomial of $A^{-1}$ equals the degree of the minimal polynomial of $A$). [Hint: note that 0 is not an eigenvalue of $A$ and hence such polynomials (minimum polynomials) have nonzero constant, assume that $m_{A^{-1}}(x)) = x^k + ... + a_1x + a_0(a_0 \neq 0)$ and $k < deg(m_A(x))$ , then reach a contradiction].

(iii) Let $A$ be a $5 \times 5$ matrix such that $m_A(x) = C_A(X) = (x+2)^3(x-2)^2$. Find the Jordan-form of the matrix $A + 4I_4$ (justify your answer).

(iv) Let $A = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}$. Find $C_A(x)$. Find $m_A(x)$. Find $dim(E_2)$ and $dim(E_3)$.

(v) Let $A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & -9 & 0 \\ 0 & 1 & 6 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$. Find $m_A(x)$ and then find the Jordan-form of $A$. For each eigenvalue $\alpha$ of $A$ find $dim(E_\alpha)$.

(vi) Let $f(x), g(x) \in C[0, 1]$. Prove that $\int_0^1 f(x)g(x)\, dx \leq \sqrt{\int_0^1 f(x)^2\, dx \int_0^1 g(x)^2\, dx}$

(vii) Give me two matrices $4 \times 4$, say $A, B$, such that $m_A(x) = m_B(x)$ and $C_A(x) = C_B(x)$, but $A$ is not similar to $B$. Tell me clearly why $A$ is not similar to $B$.

(viii) Let $<,>$ be the normal dot product on $R^4$. Let $F = span\{(1, 1, 2, 1), (-1, -1, -1, 4)\}$. Write $F^\perp$ as a span of an orthogonal basis (recall $F^\perp$ is the set of all elements in $R^4$ such that each is orthogonal to each element of $F$).

## Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com